CHAPTER 6 REVIEW EXERCISES

6.1

1. If $f(x) = \dfrac{x^2 + 2x - 3}{x^2 - 4}$, find the following function values. If a function value does not exist, so state.

   a. $f(4)$ **b.** $f(0)$

   c. $f(-2)$ **d.** $f(-3)$

In Exercises 2–3, find the domain of the given rational function.

2. $f(x) = \dfrac{x - 6}{(x - 3)(x + 4)}$

3. $f(x) = \dfrac{x + 2}{x^2 + x - 2}$

In Exercises 4–8, simplify each rational expression. If the rational expression cannot be simplified, so state.

4. $\dfrac{5x^3 - 35x}{15x^2}$

5. $\dfrac{x^2 + 6x - 7}{x^2 - 49}$

6. $\dfrac{6x^2 + 7x + 2}{2x^2 - 9x - 5}$

7. $\dfrac{x^2 + 4}{x^2 - 4}$

8. $\dfrac{x^3 - 8}{x^2 - 4}$

In Exercises 9–15, multiply or divide as indicated.

9. $\dfrac{5x^2 - 5}{3x + 12} \cdot \dfrac{x + 4}{x - 1}$

10. $\dfrac{2x + 5}{4x^2 + 8x - 5} \cdot \dfrac{4x^2 - 4x + 1}{x + 1}$

11. $\dfrac{x^2 - 9x + 14}{x^3 + 2x^2} \cdot \dfrac{x^2 - 4}{x^2 - 4x + 4}$

12. $\dfrac{1}{x^2 + 8x + 15} \div \dfrac{3}{x + 5}$

13. $\dfrac{x^2 + 16x + 64}{2x^2 - 128} \div \dfrac{x^2 + 10x + 16}{x^2 - 6x - 16}$

14. $\dfrac{y^2 - 16}{y^3 - 64} \div \dfrac{y^2 - 3y - 18}{y^2 + 5y + 6}$

15. $\dfrac{x^2 - 4x + 4 - y^2}{2x^2 - 11x + 15} \cdot \dfrac{x^4 y}{x - 2 + y} \div \dfrac{x^3 y - 2x^2 y - x^2 y^2}{3x - 9}$

16. Deer are placed into a newly acquired habitat. The deer population over time is modeled by a rational function whose graph is shown in the figure. Use the graph in the next column to answer each of the following questions.

   a. How many deer were introduced into the habitat?

   b. What is the population after 10 years?

c. What is the equation of the horizontal asymptote shown in the figure? What does this mean in terms of the deer population?

Deer Population over Time
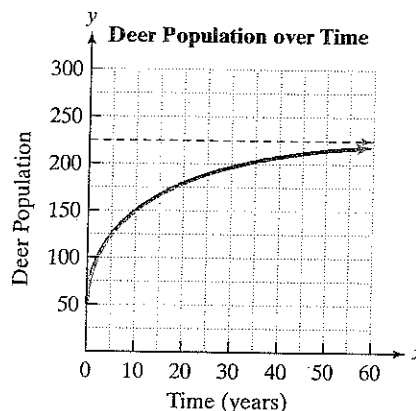
6.2

In Exercises 17–19, add or subtract as indicated. Simplify the result, if possible.

17. $\dfrac{4x + 1}{3x - 1} + \dfrac{8x - 5}{3x - 1}$

18. $\dfrac{2x - 7}{x^2 - 9} - \dfrac{x - 4}{x^2 - 9}$

19. $\dfrac{4x^2 - 11x + 4}{x - 3} - \dfrac{x^2 - 4x + 10}{x - 3}$

In Exercises 20–21, find the least common denominator of the rational expressions.

20. $\dfrac{7}{9x^3}$ and $\dfrac{5}{12x}$

21. $\dfrac{x + 7}{x^2 + 2x - 35}$ and $\dfrac{x}{x^2 + 9x + 14}$

In Exercises 22–28, perform the indicated operations. Simplify the result, if possible.

22. $\dfrac{1}{x} + \dfrac{2}{x - 5}$

23. $\dfrac{2}{x^2 - 5x + 6} + \dfrac{3}{x^2 - x - 6}$

24. $\dfrac{x - 3}{x^2 - 8x + 15} + \dfrac{x + 2}{x^2 - x - 6}$

25. $\dfrac{3x^2}{9x^2 - 16} - \dfrac{x}{3x + 4}$

26. $\dfrac{y}{y^2 + 5y + 6} - \dfrac{2}{y^2 + 3y + 2}$

27. $\dfrac{x}{x + 3} + \dfrac{x}{x - 3} - \dfrac{9}{x^2 - 9}$

28. $\dfrac{3x^2}{x - y} + \dfrac{3y^2}{y - x}$

6.3 *In Exercises 29–34, simplify each complex rational expression.*

29. $\dfrac{\dfrac{3}{x} - 3}{\dfrac{8}{x} - 8}$

30. $\dfrac{\dfrac{5}{x} + 1}{1 - \dfrac{25}{x^2}}$

31. $\dfrac{3 - \dfrac{1}{x+3}}{3 + \dfrac{1}{x+3}}$

32. $\dfrac{\dfrac{4}{x+3}}{\dfrac{2}{x-2} - \dfrac{1}{x^2 + x - 6}}$

33. $\dfrac{\dfrac{2}{x^2 - x - 6} + \dfrac{1}{x^2 - 4x + 3}}{\dfrac{3}{x^2 + x - 2} - \dfrac{2}{x^2 + 5x + 6}}$

34. $\dfrac{x^{-2} + x^{-1}}{x^{-2} - x^{-1}}$

6.4 *In Exercises 35–36, divide the polynomial by the monomial.*

35. $(15x^3 - 30x^2 + 10x - 2) \div (5x^2)$

36. $(36x^4 y^3 + 12x^2 y^3 - 60x^2 y^2) \div (6xy^2)$

In Exercises 37–40, divide as indicated.

37. $(6x^2 - 5x + 5) \div (2x + 3)$

38. $(10x^3 - 26x^2 + 17x - 13) \div (5x - 3)$

39. $(x^6 + 3x^5 - 2x^4 + x^2 - 3x + 2) \div (x - 2)$

40. $(4x^4 + 6x^3 + 3x - 1) \div (2x^2 + 1)$

6.5 *In Exercises 41–43, divide using synthetic division.*

41. $(4x^3 - 3x^2 - 2x + 1) \div (x + 1)$

42. $(3x^4 - 2x^2 - 10x - 20) \div (x - 2)$

43. $(x^4 + 16) \div (x + 4)$

In Exercises 44–45, use synthetic division and the Remainder Theorem to find the indicated function value.

44. $f(x) = 2x^3 - 5x^2 + 4x - 1;\ f(2)$

45. $f(x) = 3x^4 + 7x^3 + 8x^2 + 2x + 4;\ f\left(-\frac{1}{3}\right)$

In Exercises 46–47, use synthetic division to determine whether or not the number given to the right of each equation is a solution of

48. Use synthetic division to show that $\frac{1}{2}$ is a solution of
$$6x^3 + x^2 - 4x + 1 = 0.$$
Then solve the polynomial equation.

6.6 *In Exercises 49–55, solve each rational equation. If an equation has no solution, so state.*

49. $\dfrac{3}{x} + \dfrac{1}{3} = \dfrac{5}{x}$

50. $\dfrac{5}{3x + 4} = \dfrac{3}{2x - 8}$

51. $\dfrac{1}{x - 5} - \dfrac{3}{x + 5} = \dfrac{6}{x^2 - 25}$

52. $\dfrac{x + 5}{x + 1} - \dfrac{x}{x + 2} = \dfrac{4x + 1}{x^2 + 3x + 2}$

53. $\dfrac{2}{3} - \dfrac{5}{3x} = \dfrac{1}{x^2}$

54. $\dfrac{2}{x - 1} = \dfrac{1}{4} + \dfrac{7}{x + 2}$

55. $\dfrac{2x + 7}{x + 5} - \dfrac{x - 8}{x - 4} = \dfrac{x + 18}{x^2 + x - 20}$

56. The function
$$f(x) = \frac{4x}{100 - x}$$
models the cost, $f(x)$, in millions of dollars, to remove $x\%$ of pollutants from a river due to pesticide runoff from area farms. What percentage of the pollutants can be removed for \$16 million?

6.7 *In Exercises 57–61, solve each formula for the specified variable.*

57. $P = \dfrac{R - C}{n}$ for C

58. $\dfrac{P_1 V_1}{T_1} = \dfrac{P_2 V_2}{T_2}$ for T_1

59. $T = \dfrac{A - P}{Pr}$ for P

60. $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ for R

61. $I = \dfrac{nE}{R + nr}$ for n

62. A company is planning to manufacture affordable graphing calculators. Fixed monthly cost will be \$50,000, and it will cost \$25 to produce each calculator.

a. Write the cost function, C, of producing x graphing calculators.

b. Write the average cost function, $\overline{C}$, of producing x graphing calculators.