Definitions and Concepts

Examples

Section 4.5 Linear Programming

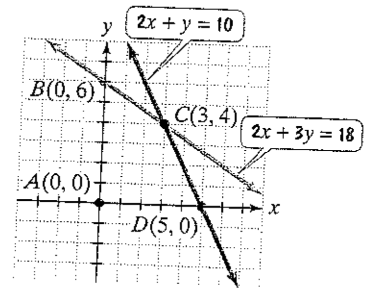
Linear programming is a method for solving problems in which a particular quantity that must be maximized or minimized is limited. An objective function is an algebraic expression in three variables modeling a quantity that must be maximized or minimized. Constraints are restrictions, expressed as linear inequalities.

Solving a Linear Programming Problem

1. Graph the system of inequalities representing the constraints.
2. Find the value of the objective function at each corner, or vertex, of the graphed region. The maximum and minimum of the objective function occur at one or more vertices.

Find the maximum value of the objective function $z = 3x + 2y$ subject to the following constraints: $x \geq 0, y \geq 0, 2x + 3y \leq 18, 2x + y \leq 10.$

1. Graph the system of inequalities representing the constraints.



2. Evaluate the objective function at each vertex.

Vertex	$z = 3x + 2y$
$A(0, 0)$	$z = 3(0) + 2(0) = 0$
$B(0, 6)$	$z = 3(0) + 2(6) = 12$
$C(3, 4)$	$z = 3(3) + 2(4) = 17$
$D(5, 0)$	$z = 3(5) + 2(0) = 15$

The maximum value of the objective function is 17.

CHAPTER 4 REVIEW EXERCISES

4.1 *In Exercises 1–3, express each interval in set-builder notation and graph the interval on a number line.*

1. $(-2, 3]$
2. $[-1.5, 2]$
3. $(-1, \infty)$

In Exercises 4–9, solve each linear inequality. Other than ∅, graph the solution set on a number line. Express the solution set in both set-builder and interval notations.

4. $-6x + 3 \leq 15$
5. $6x - 9 \geq -4x - 3$
6. $\frac{x}{3} - \frac{3}{4} - 1 > \frac{x}{2}$
7. $6x + 5 > -2(x - 3) - 25$
8. $3(2x - 1) - 2(x - 4) \geq 7 + 2(3 + 4x)$
9. $2x + 7 \leq 5x - 6 - 3x$
10. A person can choose between two charges on a checking account. The first method involves a fixed cost of \$11 per month plus 6¢ for each check written. The second method involves a fixed cost of \$4 per month plus 20¢ for each check written. How many checks should be written to make the first method a better deal?
11. A salesperson earns \$500 per month plus a commission of 20% of sales. Describe the sales needed to receive a total income that exceeds \$3200 per month.

4.2 *In Exercises 12–15, let $A = \{a, b, c\}$, $B = \{a, c, d, e\}$, and $C = \{a, d, f, g\}$. Find the indicated set.*

12. $A \cap B$
13. $A \cap C$
14. $A \cup B$
15. $A \cup C$

In Exercises 16–26, solve each compound inequality. Except for the empty set, express the solution set in both set-builder and interval notations. Graph the solution set on a number line.

16. $x \le 3$ and $x < 6$
17. $x \le 3$ or $x < 6$
18. $-2x < -12$ and $x - 3 < 5$
19. $5x + 3 \le 18$ and $2x - 7 \le -5$
20. $2x - 5 > -1$ and $3x < 3$
21. $2x - 5 > -1$ or $3x < 3$
22. $x + 1 \le -3$ or $-4x + 3 < -5$
23. $5x - 2 \le -22$ or $-3x - 2 > 4$
24. $5x + 4 \ge -11$ or $1 - 4x \ge 9$
25. $-3 < x + 2 \le 4$
26. $-1 \le 4x + 2 \le 6$
27. To receive a B in a course, you must have an average of at least 80% but less than 90% on five exams. Your grades on the first four exams were 95%, 79%, 91%, and 86%. What range of grades on the fifth exam will result in a B for the course? Use interval notation to express this range.

4.3

In Exercises 28–31, find the solution set for each equation.

28. $|2x + 1| = 7$
29. $|3x + 2| = -5$
30. $2|x - 3| - 7 = 10$
31. $|4x - 3| = |7x + 9|$

In Exercises 32–36, solve and graph the solution set on a number line. Except for the empty set, express the solution set in both set-builder and interval notations.

32. $|2x + 3| \le 15$
33. $\left|\frac{2x + 6}{3}\right| > 2$
34. $|2x + 5| - 7 < -6$
35. $-4|x + 2| + 5 \le -7$
36. $|2x - 3| + 4 \le -10$
37. Approximately 90% of the population sleeps h hours daily, where h is modeled by the inequality $|h - 6.5| \le 1$. Write a sentence describing the range for the number of hours that most people sleep. Do *not* use the phrase "absolute value" in your description.

4.4

In Exercises 38–43, graph each inequality in a rectangular coordinate system.

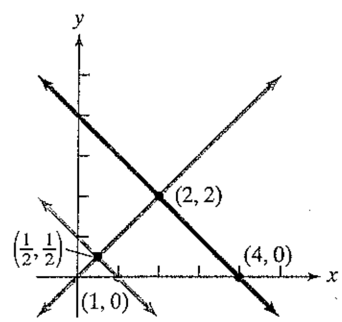
38. $3x - 4y > 12$
39. $x - 3y \le 6$
40. $y \le -\frac{1}{2}x + 2$
41. $y > \frac{3}{5}x$
42. $x \le 2$
43. $y > -3$

In Exercises 44–52, graph the solution set of each system of inequalities or indicate that the system has no solution.

44. $2x - y \le 4$, $x + y \ge 5$
45. $y < -x + 4$, $y > x - 4$
46. $-3 \le x < 5$
47. $-2 < y \le 6$
48. $x \ge 3$, $y \le 0$
49. $2x - y > -4$, $x \ge 0$
50. $x + y \le 6$, $y \ge 2x - 3$
51. $3x + 2y \ge 4$, $x - y \le 3$, $x \ge 0, y \ge 0$
52. $2x - y > 2$, $2x - y < -2$

4.5

53. Find the value of the objective function $z = 2x + 3y$ at each corner of the graphed region shown. What is the maximum value of the objective function? What is the minimum value of the objective function?



In Exercises 54–56, graph the region determined by the constraints. Then find the maximum value of the given objective function, subject to the constraints.

54. Objective Function $z = 2x + 3y$
Constraints $x \ge 0, y \ge 0$; $x + y \le 8$; $3x + 2y \ge 6$

55. Objective Function $z = x + 4y$
Constraints $0 \le x \le 5, 0 \le y \le 7$; $x + y \ge 3$

56. Objective Function $z = 5x + 6y$
Constraints $x \ge 0, y \ge 0$; $y \le x$; $2x + y \le 12$; $2x + 3y \ge 6$

57. A paper manufacturing company converts wood pulp to writing paper and newsprint. The profit on a unit of writing paper is \$500 and the profit on a unit of newsprint is \$350.
    a. Let x represent the number of units of writing paper produced daily. Let y represent the number of units of newsprint produced daily. Write the objective function that models total daily profit.