Solution Apply the facts in the box above with $(x, y) = (5, 7)$ and $r = \sqrt{x^2 + y^2} = \sqrt{(-5)^2 + 7^2} = \sqrt{74}$:

$$\sin t = \frac{y}{r} = \frac{7}{\sqrt{74}} \qquad \cos t = \frac{x}{r} = \frac{-5}{\sqrt{74}} \qquad \tan t = \frac{y}{x} = \frac{7}{-5} = -\frac{7}{5}. \quad ■$$

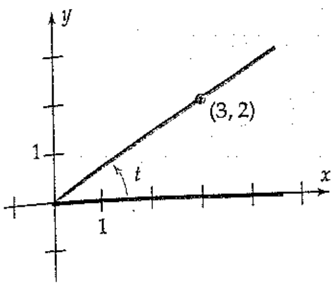


Figure 6–22

Example 6 The terminal side of a first-quadrant angle of t radians in standard position lies on the line with equation $2x - 3y = 0$. Evaluate the three trigonometric functions at t.

Solution Verify that the point (3, 2) satisfies the equation and hence lies on the terminal side of the angle (Figure 6–22). Now we have $(x, y) = (3, 2)$ and $r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 2^2} = \sqrt{13}$. Therefore,

$$\sin t = \frac{y}{r} = \frac{2}{\sqrt{13}}, \qquad \cos t = \frac{x}{r} = \frac{3}{\sqrt{13}}, \qquad \tan t = \frac{y}{x} = \frac{2/\sqrt{13}}{3/\sqrt{13}} = \frac{2}{3}. \quad ■$$

Exercises 6.2

Note: Unless stated otherwise, all angles are in standard position.

In Exercises 1–10, use the definition (not a calculator) to find the function value.

1. $\sin 3\pi/2$
2. $\sin(-\pi)$
3. $\cos 3\pi/2$
4. $\cos(-\pi/2)$
5. $\tan 4\pi$
6. $\tan(-\pi)$
7. $\cos(-3\pi/2)$
8. $\sin 9\pi/2$
9. $\cos(-11\pi/2)$
10. $\tan(-13\pi)$

In Exercises 11–14, assume that the terminal side of an angle of t radians passes through the given point on the unit circle. Find sin t, cos t, tan t.

11. $(-2/\sqrt{5}, 1/\sqrt{5})$
12. $(1/\sqrt{10}, -3/\sqrt{10})$
13. $(-3/5, -4/5)$
14. $(.6, -.8)$

In Exercises 15–29, find the exact value of the sine, cosine, and tangent of the number, without using a calculator.

15. $5\pi/6$
16. $7\pi/6$
17. $7\pi/3$
18. $17\pi/3$
19. $11\pi/4$
20. $5\pi/4$
21. $-3\pi/2$
22. 3π
23. $-23\pi/6$
24. $11\pi/6$
25. $-19\pi/3$
26. $-10\pi/3$
27. $-15\pi/4$
28. $-25\pi/4$
29. $-17\pi/2$

30. Fill the blanks in the following table. Write each entry as a fraction with denominator 2 and with a radical in the numerator. For example,

$$\sin\frac{\pi}{2} = 1 = \frac{\sqrt{4}}{2}.$$

Some students find the resulting pattern an easy way to remember these functional values.

t	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin t					
cos t					

In Exercises 31–36, write the expression as a single real number. Do not use decimal approximations.

31. $\sin(\pi/3)\cos\pi + \sin\pi\cos(\pi/3)$
32. $\sin(\pi/6)\cos(\pi/2) - \cos(\pi/6)\sin(\pi/2)$
33. $\cos(\pi/2)\cos(\pi/4) - \sin(\pi/2)\sin(\pi/4)$
34. $\cos(2\pi/3)\cos\pi + \sin(2\pi/3)\sin\pi$
35. $\sin(3\pi/4)\cos(5\pi/6) - \cos(3\pi/4)\sin(5\pi/6)$
36. $\sin(-7\pi/3)\cos(5\pi/4) + \cos(-7\pi/3)\sin(5\pi/4)$

In Exercises 37–42, find sin t, cos t, tan t when the terminal side of an angle of t radians in standard position passes through the given point.

37. (2, 7)
38. (−3, 2)
39. (−5, −6)
40. (4, −3)
41. $(\sqrt{3}, -10)$
42. $(-\pi, 2)$

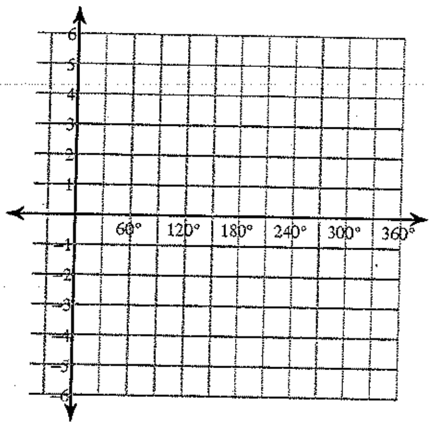
Name________________________

Graphing Trig Functions

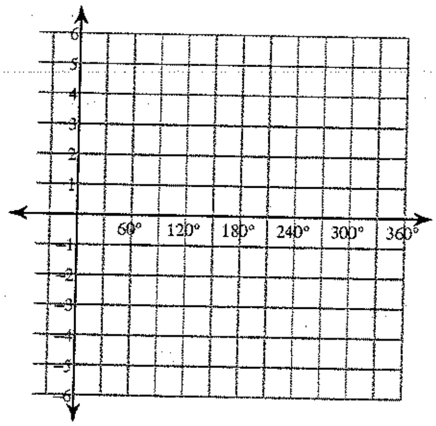
Date________________ Period____

Using degrees, find the amplitude and period of each function. Then graph.

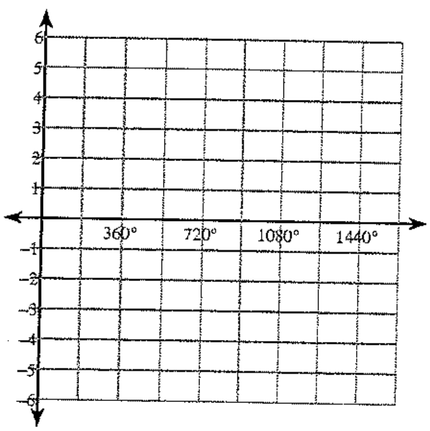
1) $y = \sin 3\theta$



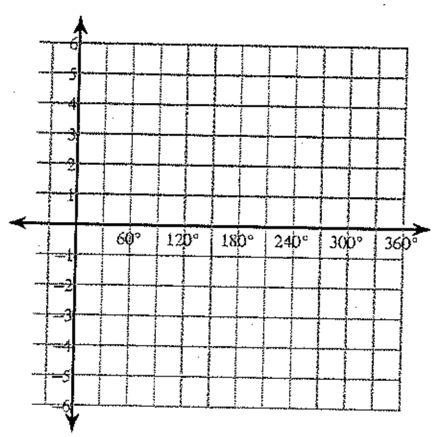
2) $y = 4\cos 3\theta$



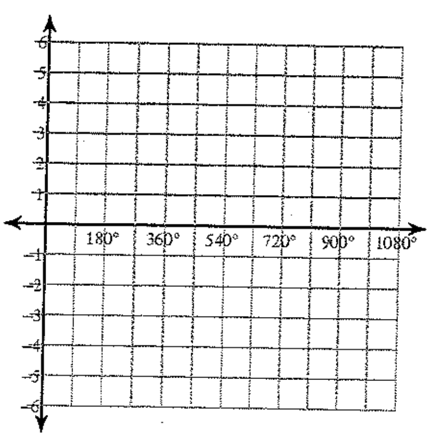
3) $y = 2\sin \frac{\theta}{3}$



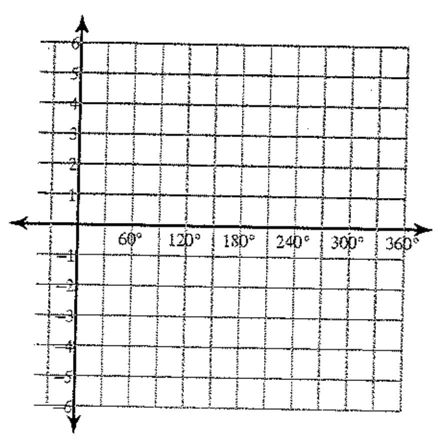
4) $y = \tan 2\theta$



5) $y = 3\cos \frac{\theta}{2}$

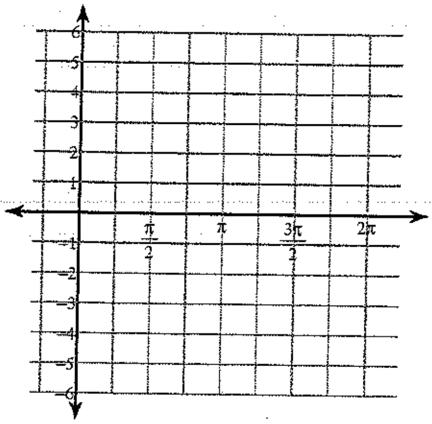


6) $y = \frac{1}{2}\tan \theta$

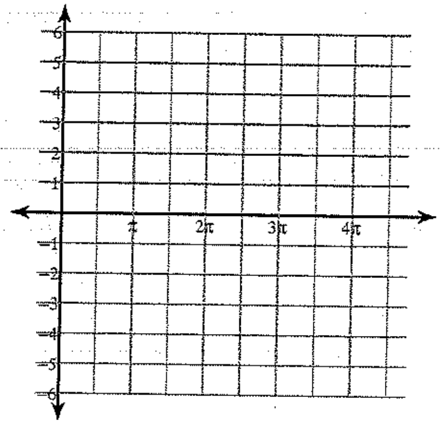


Using radians, find the amplitude and period of each function. Then graph.

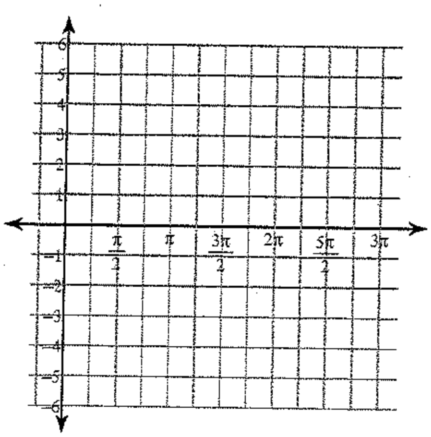
7) $y = \sin 3\theta$



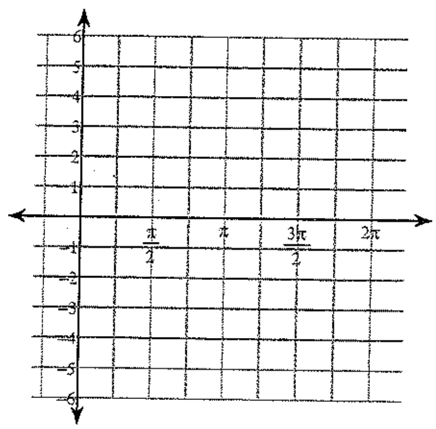
8) $y = \frac{1}{2} \tan \frac{\theta}{3}$



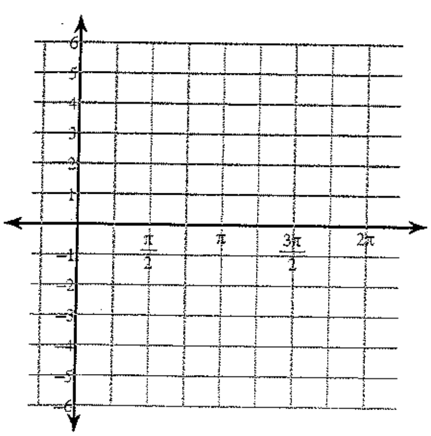
9) $y = \frac{1}{2} \sec \theta$



10) $y = 2\cos 4\theta$



11) $y = 2\csc 2\theta$



12) $y = 2\cot 2\theta$