Right triangle PQD and the Pythagorean Theorem show that $(PD)^2 = r^2 + s^2$; hence $PD = \sqrt{r^2 + s^2}$. Applying the Pythagorean Theorem to right triangle PDC, we have

$$d^2 = 3^2 + (PD)^2$$
$$d^2 = 3^2 + \left(\sqrt{r^2 + s^2}\right)^2$$
$$d^2 = 9 + r^2 + s^2$$
$$d = \sqrt{9 + r^2 + s^2}$$

HW #2 Circled problems

(c) The preceding equation expresses d in terms of r and s. By substituting $r = 45t$ and $s = 350t$ in this equation, we can express d as a function of the time t:

$$d = \sqrt{9 + r^2 + s^2}$$
$$d = \sqrt{9 + (45t)^2 + (350t)^2}$$
$$d = \sqrt{9 + 2025t^2 + 122{,}500t^2} = \sqrt{9 + 124{,}525t^2}.$$

(d) At 1:30 P.M. we have $t = 1.5$ (since noon is $t = 0$). At this time

$$d = \sqrt{9 + 124{,}525t^2} = \sqrt{9 + 124{,}525(1.5)^2} = \sqrt{280{,}190.25}$$
$$\approx 529.33 \text{ miles.} \quad \blacksquare$$

Exercises 3.5

In Exercises 1–4, find $(f + g)(x)$, $(f - g)(x)$, and $(g - f)(x)$.

1. $f(x) = -3x + 2, \quad g(x) = x^3$
2. $f(x) = x^2 + 2, \quad g(x) = -4x + 7$
3. $f(x) = 1/x, \quad g(x) = x^2 + 2x - 5$
4. $f(x) = \sqrt{x}, \quad g(x) = x^2 + 1$

In Exercises 5–8, find $(fg)(x)$, $(f/g)(x)$, and $(g/f)(x)$.

5. $f(x) = -3x + 2, \quad g(x) = x^3$
6. $f(x) = 4x^2 + x^4, \quad g(x) = \sqrt{x^2 + 4}$
7. $f(x) = x^2 - 3, \quad g(x) = \sqrt{x - 3}$
8. $f(x) = \sqrt{x^2 - 1}, \quad g(x) = \sqrt{x - 1}$

In Exercises 9–12, find the domains of fg and f/g.

9. $f(x) = x^2 + 1, \quad g(x) = 1/x$
10. $f(x) = x + 2, \quad g(x) = \dfrac{1}{x + 2}$
11. $f(x) = \sqrt{4 - x^2}, \quad g(x) = \sqrt{3x + 4}$
12. $f(x) = 3x^2 + x^4 + 2, \quad g(x) = 4x - 3$

In Exercises 13–16, find the indicated values, where $g(t) = t^2 - t$ and $f(x) = 1 + x$.

13. $g(f(0))$
14. $(f \circ g)(3)$
15. $g(f(2) + 3)$
16. $f(2g(1))$

In Exercises 17–20, find $(g \circ f)(3)$, $(f \circ g)(1)$, and $(f \circ f)(0)$.

17. $f(x) = 3x - 2, \quad g(x) = x^2$
18. $f(x) = |x + 2|, \quad g(x) = -x^2$
19. $f(x) = x, \quad g(x) = -3$
20. $f(x) = x^2 - 1, \quad g(x) = \sqrt{x}$

In Exercises 21–24, find the rule of the function $f \circ g$, the domain of $f \circ g$, the rule of $g \circ f$, and the domain of $g \circ f$.

21. $f(x) = x^2, \quad g(x) = x + 3$
22. $f(x) = -3x + 2, \quad g(x) = x^3$
23. $f(x) = 1/x, \quad g(x) = \sqrt{x}$
24. $f(x) = \dfrac{1}{2x + 1}, \quad g(x) = x^2 - 1$

In Exercises 25–28, find the rules of the functions ff and $f \circ f$.

25. $f(x) = x^3$
26. $f(x) = (x - 1)^2$
27. $f(x) = 1/x$
28. $f(x) = \dfrac{1}{x - 1}$

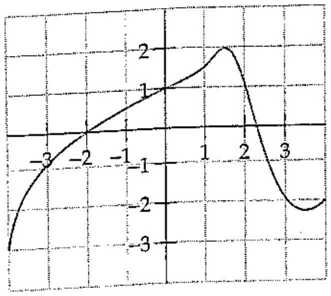
29. $f(x) = 9x + 2$, $g(x) =$ 9

30. $f(x) = \sqrt[3]{x - 1}$, $g(x) = x^3 + 1$

31. $f(x) = \sqrt[3]{x} + 2$, $g(x) = (x - 2)^3$

32. $f(x) = 2x^3 - 5$, $g(x) = \sqrt[3]{\dfrac{x + 5}{2}}$

Exercises 33 and 34 refer to the function f whose graph is shown in the figure.



33. Let g be the composite function $f \circ f$ [that is, $g(x) = (f \circ f)(x) = f(f(x))$]. Use the graph of f to fill in the following table (approximate where necessary).

x	$f(x)$	$g(x) = f(f(x))$
−4		
−3		
−2	0	1
−1		
0		
1		
2		
3		
4		

34. Use the information obtained in Exercise 33 to sketch the graph of the function g.

x	$f(x)$
1	3
2	5
3	1
4	2
5	3

t	$g(t)$
1	5
2	4
3	4
4	3
5	2

35.

x	$(g \circ f)(x)$
1	4
2	
3	5
4	
5	

36.

t	$(f \circ g)(t)$
1	
2	2
3	
4	
5	

37.

x	$(f \circ f)(x)$
1	
2	
3	3
4	
5	

38.

t	$(g \circ g)(t)$
1	
2	
3	
4	4
5	

In Exercises 39–44, write the given function as the composite of two functions, neither of which is the identity function, as in Examples 6 and 7. (There may be more than one way to do this.)

39. $f(x) = \sqrt[3]{x^2 + 2}$

40. $g(x) = \sqrt{x + 3} - \sqrt[3]{x + 3}$

41. $h(x) = (7x^3 - 10x + 17)^7$

42. $k(x) = \sqrt[3]{(7x - 3)^2}$

NOTE In many texts the inverse function of a function f is denoted f^{-1}. In this notation, for instance, the inverse of the function $f(x) = x^3 + 5$ in Example 3 would be written as $f^{-1}(x) = \sqrt[3]{x-5}$. Similarly, the reversal properties of inverse functions become

$$f^{-1}(f(x)) = x \text{ for every } x \text{ in the domain of } f; \text{ and}$$
$$f(f^{-1}(x)) = x \text{ for every } x \text{ in the domain of } f^{-1}.$$

In this context, f^{-1} does *not* mean $1/f$ (see Exercise 47).

Exercises 3.7

In Exercises 1–8, use a calculator and the Horizontal Line Test to determine whether or not the function f is one-to-one.

1. $f(x) = x^4 - 4x^2 + 3$
2. $f(x) = x^4 - 4x + 3$
3. $f(x) = x^3 + x - 5$
4. $f(x) = \begin{cases} x - 3 & \text{for } x \le 3 \\ 2x - 6 & \text{for } x > 3 \end{cases}$
5. $f(x) = x^5 + 2x^4 - x^2 + 4x - 5$
6. $f(x) = x^3 - 4x^2 + x - 10$
7. $f(x) = .1x^3 - .1x^2 - .005x + 1$
8. $f(x) = .1x^3 + .005x + 1$

In Exercises 9–22, use algebra to find the inverse of the given one-to-one function.

9. $f(x) = -x$
10. $f(x) = -x + 1$
11. $f(x) = 5x - 4$
12. $f(x) = -3x + 5$
13. $f(x) = 5 - 2x^3$
14. $f(x) = (x^5 + 1)^3$
15. $f(x) = \sqrt{4x - 7}$
16. $f(x) = 5 + \sqrt{3x - 2}$
17. $f(x) = 1/x$
18. $f(x) = 1/\sqrt{x}$
19. $f(x) = \dfrac{1}{2x + 1}$
20. $f(x) = \dfrac{x}{x + 1}$
21. $f(x) = \dfrac{x^3 - 1}{x^3 + 5}$
22. $f(x) = \sqrt[5]{\dfrac{3x - 1}{x - 2}}$

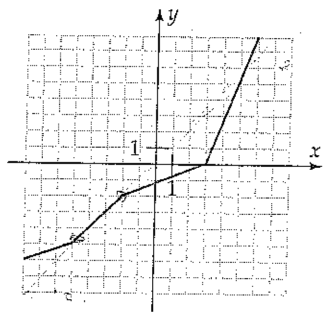
Composite Functio

In Exercises 23–28, use ~~the Round-Trip Theorem on page 191~~ to show that g is the inverse of f.

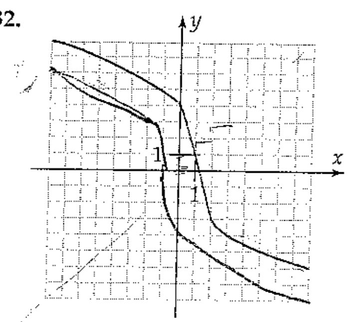
23. $f(x) = x + 1, \quad g(x) = x - 1$
24. $f(x) = 2x - 6, \quad g(x) = \dfrac{x}{2} + 3$
25. $f(x) = \dfrac{1}{x + 1}, \quad g(x) = \dfrac{1 - x}{x}$
26. $f(x) = \dfrac{-3}{2x + 5}, \quad g(x) = \dfrac{-3 - 5x}{2x}$
27. $f(x) = x^5, \quad g(x) = \sqrt[5]{x}$
28. $f(x) = x^3 - 1, \quad g(x) = \sqrt[3]{x + 1}$
29. Show that the inverse function of the function f whose rule is $f(x) = \dfrac{2x + 1}{3x - 2}$ is f itself.
30. List three different functions (other than the one in Exercise 29), each of which is its own inverse. [Many correct answers are possible.]

In Exercises 31 and 32, the graph of a function f is given. Sketch the graph of the inverse function of f. [Reflect carefully.]

31.



32.



In Exercises 33–38, each given function has an inverse function. Sketch the graph of the inverse function.

33. $f(x) = \sqrt{x+3}$

34. $f(x) = \sqrt{3x-2}$

35. $f(x) = .3x^5 + 2$

36. $f(x) = \sqrt[3]{x+3}$

37. $f(x) = \sqrt[5]{x^3 + x - 2}$

38. $f(x) = \begin{cases} x^2 - 1 & \text{for } x \le 0 \\ -.5x - 1 & \text{for } x > 0 \end{cases}$

In Exercises 39–46, none of the functions has an inverse. State at least one way of restricting the domain of the function (that is, find a function with the same rule and a smaller domain) so that the restricted function has an inverse. Then find the rule of the inverse function.

Example: *$f(x) = x^2$ has no inverse. But the function h with domain all $x \ge 0$ and rule $h(x) = x^2$ is increasing (its graph is the right half of the graph of f—see Figure 2–2 on page 60)—and therefore has an inverse.*

39. $f(x) = |x|$

40. $f(x) = |x - 3|$

41. $f(x) = -x^2$

42. $f(x) = x^2 + 4$

43. $f(x) = \dfrac{x^2 + 6}{2}$

44. $f(x) = \sqrt{4 - x^2}$

45. $f(x) = \dfrac{1}{x^2 + 1}$

46. $f(x) = 3(x + 5)^2 + 2$

47. (a) Using the f^{-1} notation for inverse functions, find $f^{-1}(x)$ when $f(x) = 3x + 2$.
~~(b) Find $f^{-1}(1)$ and $1/f(1)$. Conclude that f^{-1} is not the same function as $1/f$.~~

48. Let C be the temperature in degrees Celsius. Then the temperature in degrees Fahrenheit is given by $f(C) = \frac{9}{5}C + 32$. Let g be the function that converts degrees Fahrenheit to degrees Celsius. Show that g is the inverse function of f and find the rule of g.

Thinkers

49. Let m and b be constants with $m \neq 0$. Show that the function $f(x) = mx + b$ has an inverse function g and find the rule of g.

50. Prove that the function $f(x) = 1 - .2x^3$ of Example 1(c) is one-to-one by showing that it satisfies the definition:

If $a \neq b$, then $f(a) \neq f(b)$.

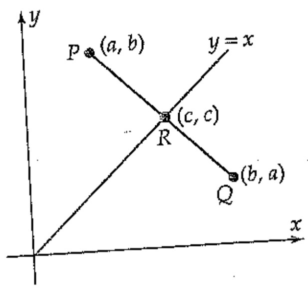
[*Hint:* Use the rule of f to show that when $f(a) = f(b)$, then $a = b$. If this is the case, then it is impossible to have $f(a) = f(b)$ when $a \neq b$.]

51. Show that the points $P = (a, b)$ and $Q = (b, a)$ are symmetric with respect to the line $y = x$ as follows.

(a) Find the slope of the line through P and Q.

(b) Use slopes to show that the line through P and Q is perpendicular to $y = x$.

(c) Let R be the point where the line $y = x$ intersects line segment PQ. Since R is on $y = x$, it has coordinates (c, c) for some number c, as shown in the figure. Use the distance formula to show that segment PR has the same length as segment RQ. Conclude that the line $y = x$ is the perpendicular bisector of segment PQ. Therefore, P and Q are symmetric with respect to the line $y = x$.



52. (a) Experiment with your calculator or use some of the preceding exercises to find four different increasing functions. For each function, sketch the graph of the function and the graphs of its inverse on the same set of axes.

(b) Based on the evidence in part (a), do you think the following statement true or false: The inverse function of every increasing function is also an increasing function.

(c) Do parts (a) and (b) with "increasing" replaced by "decreasing."

53. Prove the Round-Trip Theorem (page 191) as follows. By hypothesis, f and g have these properties:

(1) $g(f(x)) = x$ for every number x in the domain of f;

(2) $f(g(y)) = y$ for every number y in the domain of g.

(a) Prove that f is one-to-one by showing that

if $a \neq b$, then $f(a) \neq f(b)$.

[*Hint:* If $f(a) = f(b)$, apply g to both sides and use **(1)** to show that $a = b$. Consequently, if $a \neq b$, it is impossible to have $f(a) = f(b)$.]

(b) If $g(y) = x$, show that $f(x) = y$. [*Hint:* Use **(2)**.]

(c) If $f(x) = y$, show that $g(y) = x$. [*Hint:* Use **(1)**.]

Parts (b) and (c) prove that

$g(y) = x$ exactly when $f(x) = y$.

Hence, g is the inverse function of f (see page 188).