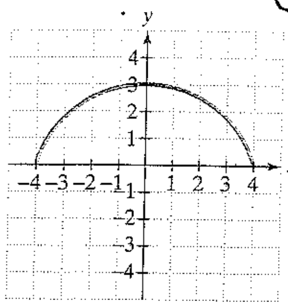
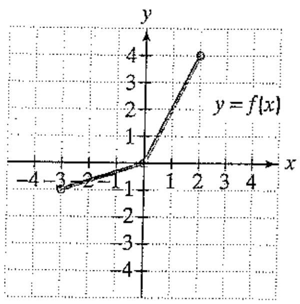
22.



23. Use the graph of f in the figure shown to draw the graph of its inverse function.



9.3 *In Exercises 24–26, write each equation in its equivalent exponential form.*

24. $\frac{1}{2} = \log_{49} 7$

25. $3 = \log_4 x$

26. $\log_3 81 = y$

In Exercises 27–29, write each equation in its equivalent logarithmic form.

27. $6^3 = 216$

28. $b^4 = 625$

29. $13^y = 874$

In Exercises 30–40, evaluate each expression without using a calculator. If evaluation is not possible, state the reason.

30. $\log_4 64$

31. $\log_5 \frac{1}{25}$

32. $\log_3(-9)$

33. $\log_{16} 4$

34. $\log_{17} 17$

35. $\log_3 3^8$

36. $\ln e^5$

37. $\log_3 \frac{1}{\sqrt{3}}$

38. $\ln \frac{1}{e^2}$

39. $\log \frac{1}{1000}$

40. $\log_3(\log_8 8)$

41. Graph $f(x) = 2^x$ and $g(x) = \log_2 x$ in the same rectangular coordinate system. Use the graphs to determine each function's domain and range.

42. Graph $f(x) = (\frac{1}{3})^x$ and $g(x) = \log_{\frac{1}{3}} x$ in the same rectangular coordinate system. Use the graphs to determine each function's domain and range.

In Exercises 43–45, find the domain of each logarithmic function.

43. $f(x) = \log_8(x + 5)$

44. $f(x) = \log(3 - x)$

45. $f(x) = \ln(x - 1)^2$

In Exercises 46–48, simplify each expression.

46. $\ln e^{6x}$

47. $e^{\ln\sqrt{x}}$

48. $10^{\log 4x^2}$

49. On the Richter scale, the magnitude, R, of an earthquake of intensity I is given by $R = \log\frac{I}{I_0}$, where I_0 is the intensity of a barely felt zero-level earthquake. If the intensity of an earthquake is $1000I_0$, what is its magnitude on the Richter scale?

50. Students in a psychology class took a final examination. As part of an experiment to see how much of the course content they remembered over time, they took equivalent forms of the exam in monthly intervals thereafter. The average score, $f(t)$, for the group after t months is modeled by the function $f(t) = 76 - 18\log(t + 1)$, where $0 \le t \le 12$.

a. What was the average score when the exam was first given?

b. What was the average score, to the nearest tenth, after 2 months? 4 months? 6 months? 8 months? one year?

c. Use the results from parts (a) and (b) to graph f. Describe what the shape of the graph indicates in terms of the material retained by the students.

51. The formula

$$t = \frac{1}{c}\ln\left(\frac{A}{A - N}\right)$$

describes the time, t, in weeks, that it takes to achieve mastery of a portion of a task. In the formula, A represents maximum learning possible, N is the portion of the learning that is to be achieved, and c is a constant used to measure an individual's learning style. A 50-year-old man decides to start running as a way to maintain good health. He feels that the maximum rate he could ever hope to achieve is 12 miles per hour. How many weeks will it take before the man can run 5 miles per hour if $c = 0.06$ for this person?

9.4 *In Exercises 52–55, use properties of logarithms to expand each logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator. Assume that all variables represent positive numbers.*

52. $\log_6(36x^3)$

53. $\log_4\left(\frac{\sqrt{x}}{64}\right)$

54. $\log_2\left(\frac{xy^2}{64}\right)$

55. $\ln\sqrt[3]{\frac{x}{e}}$

In Exercises 56–59, use properties of logarithms to condense each logarithmic expression. Write the expression as a single logarithm whose coefficient is 1.

56. $\log_b 7 + \log_b 3$

57. $\log 3 - 3\log x$

58. $3\ln x + 4\ln y$

59. $\frac{1}{2}\ln x - \ln y$

In Exercises 60–61, use common logarithms or natural logarithms and a calculator to evaluate to four decimal places.

60. $\log_6 72{,}348$

61. $\log_4 0.863$

In Exercises 62–65, determine whether each equation is true or false. Where possible, show work to support your conclusion. If the statement is false, make the necessary change(s) to produce a true statement.

62. $(\ln x)(\ln 1) = 0$

63. $\log(x + 9) - \log(x + 1) = \frac{\log(x + 9)}{\log(x + 1)}$

64. $(\log_2 x)^4 = 4\log_2 x$

65. $\ln e^x = x\ln e$

9.5 *In Exercises 66–71, solve each exponential equation. Where necessary, express the solution set in terms of natural logarithms and use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.*

66. $2^{4x-2} = 64$

67. $125^x = 25$

68. $9^x = \frac{1}{27}$

69. $8^x = 12{,}143$

70. $9e^{5x} = 1269$

71. $30e^{0.045x} = 90$

In Exercises 72–81, solve each logarithmic equation.

72. $\log_5 x = -3$

73. $\log x = 2$

74. $\log_4(3x - 5) = 3$

75. $\ln x = -1$

76. $3 + 4\ln(2x) = 15$

77. $\log_2(x + 3) + \log_2(x - 3) = 4$

78. $\log_3(x - 1) - \log_3(x + 2) = 2$

79. $\log_4(3x - 5) = \log_4 3$

80. $\ln(x + 4) - \ln(x + 1) = \ln x$

81. $\log_6(2x + 1) = \log_6(x - 3) + \log_6(x + 5)$

82. The function $P(x) = 14.7e^{-0.21x}$ models the average atmospheric pressure, $P(x)$, in pounds per square inch, at an altitude of x miles above sea level. The atmospheric pressure at the peak of Mt. Everest, the world's highest mountain, is 4.6 pounds per square inch. How many miles above sea level, to the nearest tenth of a mile, is the peak of Mt. Everest?

83. The amount of carbon dioxide in the atmosphere, measured in parts per million, has been increasing as a result of the burning of oil and coal. The buildup of gases and particles traps heat and raises the planet's temperature, a phenomenon called the greenhouse effect. Carbon dioxide accounts for about half of the warming. The function $f(t) = 364(1.005)^t$ projects carbon dioxide concentration, $f(t)$, in parts per million, t years after 2000. Using the projections given by the function, when will the carbon dioxide concentration be double the preindustrial level of 280 parts per million?

84. The function $W(x) = 0.37\ln x + 0.05$ models the average walking speed, $W(x)$, in feet per second, of residents in a city whose population is x thousand. Visitors to New York City frequently feel they are moving too slowly to keep pace with New Yorkers' average walking speed of 3.38 feet per second. What is the population of New York City? Round to the nearest thousand.

85. Use the compound interest formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

to solve this problem. How long, to the nearest tenth of a year, will it take \$12,500 to grow to \$20,000 at 6.5% annual interest compounded quarterly?

Use the compound interest formula

$$A = Pe^{rt}$$

to solve Exercises 86–87.

86. How long, to the nearest tenth of a year, will it take \$50,000 to triple in value at 7.5% annual interest compounded continuously?

87. What interest rate is required for an investment subject to continuous compounding to triple in 5 years?

9.6

88. According to the U.S. Bureau of the Census, in 1990 there were 22.4 million residents of Hispanic origin living in the United States. By 2005 the number had increased to 41.9 million. The exponential growth function $A = 22.4e^{kt}$ describes the U.S. Hispanic population, A, in millions, t years after 1990.
   a. Find k, correct to three decimal places.
   b. Use the resulting model to project the Hispanic resident population in 2010.
   c. In which year will the Hispanic resident population reach 60 million?

89. Use the exponential decay model, $A = A_0e^{kt}$, to solve this exercise. The half-life of polonium-210 is 140 days. How long will it take for a sample of this substance to decay to 20% of its original amount?